Indian Forest Service
(Main) Exam, 2020

JKLO–B–MTH

MATHEMATICS

PAPER—II

Time Allowed : Three Hours	Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in ENGLISH only.

SECTION—A

1. (a) Let p be a prime number. Then show that

$$(p-1)!+1 \equiv 0 \bmod (p)$$

Also, find the remainder when $6^{44} \cdot (22)!+3$ is divided by 23. 8

(b) (i) If $u = u(y-z,\ z-x,\ x-y)$, then find the value of $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

(ii) If $u(x, y, z) = \frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}$, then find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$. 8

(c) Evaluate the integral $\iint\limits_R (x-y)^2 \cos^2(x+y)\,dx\,dy$, where R is the rhombus with successive vertices at $(\pi, 0)$, $(2\pi, \pi)$, $(\pi, 2\pi)$ and $(0, \pi)$. 8

(d) Solve graphically the following LPP :

$$\text{Max} \quad z = 5x_1 - 3x_2$$

subject to

$$3x_1 + 2x_2 \le 12$$
$$-x_1 + x_2 \ge 1$$
$$-x_1 + x_2 \le 2$$
$$x_1, x_2 \ge 0$$

If the objective function z is changed to Max $z = 6x_1 + 4x_2$, while the constraints remain the same, then comment on the number of solutions. Will (4, 0) be also a solution? 8

(e) Evaluate the integral $\int\limits_C \text{Re}(z^2)\,dz$ from 0 to $2+4i$ along the curve $C: y = x^2$. 8

2. (a) Let R be a non-zero commutative ring with unity. Show that M is a maximal ideal in a ring R if and only if R/M is a field. 10

(b) Show that the sequence of functions $\{f_n(x)\}$, where $f_n(x) = nx(1-x)^n$, does not converge uniformly on [0, 1]. 15

(c) Using Cauchy theorem and Cauchy integral formula, evaluate the integral

$$\oint_C \frac{e^z}{z^2(z+1)^3}\,dz$$

where C is $|z|=2$. 15

3. (a) Find the extreme values of $f(x, y, z) = 2x + 3y + z$ such that $x^2 + y^2 = 5$ and $x + z = 1$. 10

(b) Let G be a finite group and let p be a prime. If p^m divides order of G, then show that G has a subgroup of order p^m, where m is a positive integer. 15

(c) Solve the following LPP by simplex method :

$$\text{Max} \quad z = 2x_1 + x_2$$

subject to

$$\begin{aligned} 2x_1 - 2x_2 &\le 1 \\ 2x_1 - 4x_2 &\le 3 \\ 2x_1 + x_2 &\le 2 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Does there exist an alternate optimal solution? If yes, give one and hence find all the optimal solutions. 15

4. (a) Show that the bilinear transformation

$$w = e^{i\theta_0}\left(\frac{z - z_0}{z - \bar{z}_0}\right)$$

z_0 being in the upper half of the z-plane, maps the upper half of the z-plane into the interior of the unit circle in the w-plane. If under this transformation, the point $z = i$ is mapped into $w = 0$ while the point at infinity is mapped into $w = -1$, then find this transformation. 10

(b) Let K be a finite field. Show that the number of elements in K is p^n, where p is a prime, which is characteristic of K and $n \ge 1$ is an integer. Also, prove that $\dfrac{\mathbb{Z}_3[X]}{(X^2+1)}$ is a field. How many elements does this field have? 15

(c) Find the minimum transportation cost using Vogel's approximation method for the following transportation problem :

Destinations

		D_1	D_2	D_3	D_4	*Availability*
	S_1	9	16	15	9	15
Sources	S_2	2	1	3	5	25
	S_3	6	4	7	3	20
Demand		10	15	25	10	

15

SECTION—B

5. (a) Construct a partial differential equation of all surfaces of revolution having the z-axis as the axis of rotation. 8

(b) Using Newton-Raphson method, find the value of $(37)^{1/3}$, correct to four decimal places. 8

(c) Answer the following questions :

(i) Convert $(14231)_8$ into an equivalent binary number and then find the equivalent decimal number.

(ii) Convert $(43503)_{10}$ into an equivalent binary number and then find the equivalent hexadecimal number. 8

(d) Find the condition on a, b, c (real numbers) such that the dynamical system with equations $\dot{p} = aq - q^2$, $\dot{q} = bp + cq$ is Hamiltonian. Compute also the Hamiltonian of the system. 8

(e) Find the general solution of the partial differential equation

$$p \tan x + q \tan y = \tan z$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 8

6. (a) Find the general solution and singular solution of the partial differential equation

$$6yz - 6pxy - 3qy^2 + pq = 0$$

10+5=15

(b) Find the Lagrange interpolating polynomial that fits the following data values :

x :	−1	2	3	5
$f(x)$:	−1	10	25	60

Also, interpolate at $x = 2{\cdot}5$, correct to three decimal places. 15

(c) In a fluid flow, the velocity vector is given by $\vec{V} = 2x\vec{i} + 3y\vec{j} - 5z\vec{k}$. Determine the equation of the streamline passing through a point $A = (4, 8, 1)$. 10

7. *(a)* Write down the algorithm and flowchart for solving numerically the differential equation $\frac{dy}{dx} = f(x, y) = 1 + x\cos y$ with initial condition : at $x = x_0$, $y = y_0$ and step length h by Euler's method up to $x = x_n = x_0 + nh$. 7+8=15

(b) In a two-dimensional fluid flow, the velocity components are given by $u = x - ay$ and $v = -ax - y$, where a is constant. Show that the velocity potential exists for this flow and determine the appropriate velocity potential. Also, determine the corresponding stream function that would represent the flow. 15

(c) Find the solution of the following differential equation :

$$2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = ye^x$$ 10

8. *(a)* A particle is attracted to a center by a force which varies inversely as the cube of its distance from the center. Identify the generalized coordinates and write down the Lagrangian of the system. Derive then the equations of motion and solve them for the orbits. Discuss how the nature of orbits depends on the parameters of the system. 20

(b) Evaluate the integral $\int_0^2 \frac{x}{1+x^3}\, dx$, using trapezoidal rule with $h = \frac{1}{4}$, correct to three decimal places. (h is the length of subinterval) 10

(c) Solve the following system of linear equations using Gaussian elimination method :

$$5x_1 + 2x_2 + x_3 = -2$$
$$6x_1 + 3x_2 + 2x_3 = 1$$
$$x_1 - x_2 + 2x_3 = 0$$ 10

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