I.F.S. EXAM-(M) 2018

FSI–D–MTH

MATHEMATICS

PAPER—II

Time Allowed : Three Hours	Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections **A** and **B**.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Answers must be written in **ENGLISH** only.

SECTION—A

1. *(a)* Prove that a non-commutative group of order $2n$, where n is an odd prime, must have a subgroup of order n. 8

(b) A function $f : [0, 1] \to [0, 1]$ is continuous on $[0, 1]$. Prove that there exists a point c in $[0, 1]$ such that $f(c) = c$. 10

(c) If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine v so that $u + iv$ is a regular function of $x + iy$. 10

(d) Solve by simplex method the following Linear Programming Problem : 12

$$\text{Maximize } Z = 3x_1 + 2x_2 + 5x_3$$

subject to the constraints

$$\begin{aligned} x_1 + 2x_2 + x_3 &\le 430 \\ 3x_1 + 2x_3 &\le 460 \\ x_1 + 4x_2 &\le 420 \\ x_1,\ x_2,\ x_3 &\ge 0 \end{aligned}$$

2. *(a)* Find all the homomorphisms from the group $(\mathbb{Z}, +)$ to $(\mathbb{Z}_4, +)$. 10

(b) Consider the function f defined by

$$f(x, y) = \begin{cases} xy\,\dfrac{x^2 - y^2}{x^2 + y^2}, & \text{where } x^2 + y^2 \ne 0 \\ 0, & \text{where } x^2 + y^2 = 0 \end{cases}$$

Show that $f_{xy} \ne f_{yx}$ at $(0, 0)$. 10

(c) Prove that $\int_0^\infty \cos x^2 dx = \int_0^\infty \sin x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$. 10

(d) Let R be a commutative ring with unity. Prove that an ideal P of R is prime if and only if the quotient ring R/P is an integral domain. 10

3. *(a)* Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. 10

(b) Show by an example that in a finite commutative ring, every maximal ideal need not be prime. 10

(c) Evaluate the integral $\int_0^{2\pi} \cos^{2n}\theta\, d\theta$, where n is a positive integer. 10

(d) Show that the improper integral $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}}\, dx$ is convergent. 10

4. *(a)* Show that

$$\iint_R x^{m-1} y^{n-1} (1-x-y)^{l-1} \, dx \, dy = \frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n)}; \quad l, m, n > 0$$

taken over R : the triangle bounded by $x=0$, $y=0$, $x+y=1$. 10

(b) Let $f_n(x) = \dfrac{x}{n+x^2}$, $x \in [0, 1]$. Show that the sequence $\{f_n\}$ is uniformly convergent on $[0, 1]$. 8

(c) Let H be a cyclic subgroup of a group G. If H be a normal subgroup of G, prove that every subgroup of H is a normal subgroup of G. 10

(d) The capacities of three production facilities S_1, S_2 and S_3 and the requirements of four destinations D_1, D_2, D_3 and D_4 and transportation costs in rupees are given in the following table :

	D_1	D_2	D_3	D_4	*Capacity*
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Find the minimum transportation cost using Vogel's Approximation Method (VAM). 12

SECTION—B

5. *(a)* Find the partial differential equation of all planes which are at a constant distance a from the origin. 10

(b) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the line $x=0$ and a curve through the points with the following coordinates :

x	0·0	0·25	0·50	0·75	1·00	1·25	1·50
y	1·0	0·9896	0·9589	0·9089	0·8415	0·8029	0·7635

Estimate the volume of the solid formed using Weddle's rule. 10

(c) Write a program in BASIC to multiply two matrices (checking for consistency for multiplication is required). 10

(d) Air, obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if ρ be the density and v be the velocity at a distance x from a fixed point at time t, then $\dfrac{\partial^2 \rho}{\partial t^2} = \dfrac{\partial^2}{\partial x^2} \{\rho(v^2 + k)\}$. 10

6. (a) Find the complete integral of the partial differential equation $(p^2 + q^2)x = zp$ and deduce the solution which passes through the curve $x = 0$, $z^2 = 4y$. Here $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. 12

(b) Apply fourth-order Runge-Kutta method to compute y at $x = 0 \cdot 1$ and $x = 0 \cdot 2$, given that $\frac{dy}{dx} = x + y^2$, $y = 1$ at $x = 0$. 12

(c) For a particle having charge q and moving in an electromagnetic field, the potential energy is $U = q(\phi - \vec{v} \cdot \vec{A})$, where ϕ and $\vec{A}$ are, respectively, known as the scalar and vector potentials. Derive expression for Hamiltonian for the particle in the electromagnetic field. 8

(d) Write a program in BASIC to implement trapezoidal rule to compute $\int_0^{10} e^{-x^2} dx$ with 10 subdivisions. 8

7. (a) Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

If the solution of the above equation represents a sphere, what will be the coordinates of its centre? 8

(b) The velocity v (km/min) of a moped is given at fixed interval of time (min) as below :

t	0·1	0·2	0·3	0·4	0·5	0·6
v	1·00	1·104987	1·219779	1·34385	1·476122	1·615146

t	0·7	0·8	0·9	1·0	1·1
v	1·758819	1·904497	2·049009	2·18874	2·31977

Estimate the distance covered during the time (use Simpson's one-third rule). 10

(c) Assuming a 16-bit computer representation of signed integers, represent –44 in 2's complement representation. 10

(d) In the case of two-dimensional motion of a liquid streaming past a fixed circular disc, the velocity at infinity is u in a fixed direction, where u is a variable. Show that the maximum value of the velocity at any point of the fluid is $2u$. Prove that the force necessary to hold the disc is $2m\dot{u}$, where m is the mass of the liquid displaced by the disc. 12

8. (a) Find a real function V of x and y, satisfying $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$ and reducing to zero, when $y = 0$. 10

(b) The equation $x^6 - x^4 - x^3 - 1 = 0$ has one real root between 1·4 and 1·5. Find the root to four places of decimal by regula-falsi method. 10

(c) A particle of mass m is constrained to move on the inner surface of a cone of semi-angle α under the action of gravity. Write the equation of constraint and mention the generalized coordinates. Write down the equation of motion. 10

(d) Two sources, each of strength m, are placed at the points $(-a, 0)$, $(a, 0)$ and a sink of strength $2m$ at the origin. Show that the streamlines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$, where λ is a variable parameter.

Show also that the fluid speed at any point is $(2ma^2)/(r_1 r_2 r_3)$, where r_1, r_2, r_3 are the distances of the point from the sources and the sink. 10

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