PHYSICS

PAPER—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections **A** and **B**.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Answers must be written in **ENGLISH** only.

Useful Constants :

Electron charge (e)	$= 1 \cdot 602 \times 10^{-19}$ C
Electron rest mass (m_e)	$= 9 \cdot 109 \times 10^{-31}$ kg
Proton mass (m_p)	$= 1 \cdot 672 \times 10^{-27}$ kg
Vacuum permittivity (ε_0)	$= 8 \cdot 854 \times 10^{-12}$ farad/m
Vacuum permeability (μ_0)	$= 1 \cdot 257 \times 10^{-6}$ henry/m
Velocity of light in free space (c)	$= 3 \times 10^{8}$ m / s
Boltzmann constant (k)	$= 1 \cdot 380 \times 10^{-23}$ J / K
Electronvolt (eV)	$= 1 \cdot 602 \times 10^{-19}$ J
Planck constant (h)	$= 6 \cdot 626 \times 10^{-34}$ J s
Stefan constant (σ)	$= 5 \cdot 67 \times 10^{-8}$ W m^{-2} K^{-4}
Avogadro number (N)	$= 6 \cdot 022 \times 10^{26}$ kmol^{-1}
Gas constant (R)	$= 8 \cdot 31 \times 10^{3}$ J kmol^{-1} K^{-1}
exp (1)	$= 2 \cdot 718$

SECTION—A

1. *(a)* Construct the Hamiltonian of a charged particle with charge q and mass m moving with the velocity $\vec{v}$ in the external electromagnetic field, $\vec{E} = E_0\hat{i}$, $\vec{B} = B_0\hat{k}$, where E_0 and B_0 are constants. 8

   (b) Calculate the first relativistic correction to the kinetic energy of a particle with rest mass m_0 and speed v. 8

   (c) Calculate the focal lengths of the components of an achromatic telescope objective having a power of $+3 \cdot 333$ diopter, made from crown glass and flint glass, whose dispersive powers are $0 \cdot 012$ and $0 \cdot 02$ respectively. 8

   (d) A particle moving in a central force field located at $r = 0$, describes a spiral, $r = e^{-\theta}$. Find the force law. 8

   (e) Thin transparent films produce colours when a white light falls on them or they act as antireflecting coatings. Find the conditions for these characteristics. 8

2. *(a)* A particle of mass m is constrained to move on a plane curve $xy = c$ with $c > 0$, under gravity, where y-axis is vertical. Construct the Lagrangian of the system and obtain the Euler-Lagrange equation of motion. 10

   (b) Consider a homogeneous cube of total mass M and side a. Taking the origin at one corner of the cube and axes along the edges of the cube, construct the moment of inertia tensor. Calculate principal moment of inertia. 15

   (c) *(i)* Give the equation of motions for a particle executing simple harmonic motion for undamped, damped and forced vibrations, and explain the terms.

   (ii) Give a plot between the displacement and time for each case of the above and interpret.

   (iii) What are the conditions for critical damping and resonance? 8+4+3=15

3. *(a)* In special theory of relativity, the Hamiltonian of a free particle with rest mass m_0 is given by $\sqrt{p^2c^2 + m_0^2c^4}$. Obtain the Lagrangian of the system using Legendre transformation. 10

   (b) Show that the wave equation for the propagation of electromagnetic scalar potential $\phi(x, y, z, t)$

$$\left[\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\phi(x, y, z, t) = 0$$

   remains invariant under Lorentz transformation. 10

(c) (i) When a monochromatic light passes through a single slit, dark and bright fringes are observed on a screen kept far away from the slit. Draw an amplitude and intensity pattern of the light on the screen.

(ii) Write an expression for intensity distribution and give the conditions for bright and dark fringes.

(iii) If the slit width is $0 \cdot 3$ mm and the distance between the screen and the slit is about 30 cm, then what is the angle of diffraction for the first minimum? ($\lambda = 5 \times 10^{-5}$ cm) 8+8+4=20

4. (a) (i) What is population inversion? How is it efficiently achieved in He-Ne laser? Show it with necessary energy levels diagram.

(ii) Show that for a normal optical source emitting at 600 nm at a temperature of 10^3 K, the spontaneous emission dominates over stimulated emission. 8+7=15

(b) With the help of a neat diagram, explain spherical aberration. Briefly discuss the methods to minimize spherical aberration. 10

(c) (i) How are linearly polarised and circularly polarised lights generated from unpolarised light and are analysed?

(ii) A calcite crystal is to be used for producing a quarter wave plate. The refractive indices of the crystal for ordinary ray and extraordinary ray are $1 \cdot 65836$ and $1 \cdot 48641$ respectively. What is the thickness of the plate required if the light of wavelength $632 \cdot 8$ nm is to be used? 10+5=15

SECTION—B

5. (a) There is a potential gradient of 100 V/m normal to the surface of the earth. Assuming the earth to be a charged sphere of radius 6370 km, find the total charge on the earth. 8

(b) State and explain Biot-Savart law. Obtain an expression for the magnetic field at the center of a circular loop of radius r metres, carrying a current of I amperes. 8

(c) In free space, the electric field of electromagnetic wave is given by

$$\vec{E}(x, t) = 100 \cos(\omega t - kx)\hat{y} \quad \text{volt/metre}$$

Find the average power crossing a circular area of radius 2 metres in the yz-plane. 8

(d) Consider N molecules of a gas obeying van der Waals' equation of state given by

$$\left(P + \frac{aN^2}{V^2}\right)(V - Nb) = Nk_B T$$

where a is a measure of the attractive forces between the molecules and b is another constant proportional to the size of the molecules. The other symbols have their usual meanings. Show that during an isothermal expansion from volume V_1 to volume V_2 quasi-statically and reversibly, the work done is

$$W = -Nk_B T \log\left(\frac{V_2 - Nb}{V_1 - Nb}\right) + aN^2\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$ 8

(e) Starting from the first law of thermodynamics, show that

$$C_p - C_v = \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_P$$ 8

6. (a) Obtain an expression for the electric potential and electric field strength at a point due to an electric dipole. 15

(b) (i) Show that the electric and magnetic energy densities in a plane travelling wave are equal. Also prove that the total energy density $= \varepsilon_0 E^2 = \mu_0 H^2$.

(ii) Deduce the equation of continuity based on Maxwell's equations. 10+5=15

(c) Explain Planck's formula for black-body radiation. Use this formula to show that energy radiated per unit time per unit area in the range $d\lambda$ of λ is

$$e(\lambda, T) = \left(\frac{2\pi c^2 h}{\lambda^5}\right)(e^{\beta hc/\lambda} - 1)^{-1}\, d\lambda$$ 10

7. (a) Using the set of four Maxwell's equations, obtain the Lorentz condition relation between the scalar potential ϕ and vector potential A, i.e.,

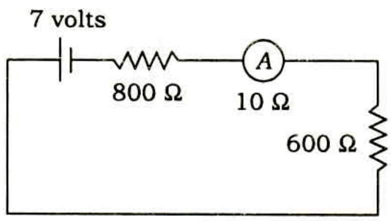
$$\nabla \cdot A + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$

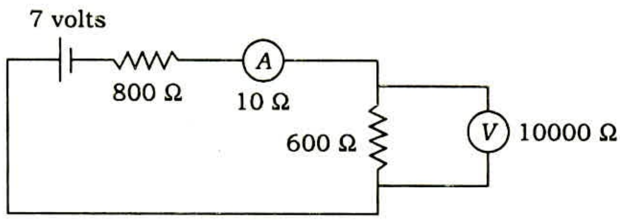
and discuss the gauge transformations, Lorentz gauge and Coulomb gauge. 15+5=20

(b) Two resistors of 600 Ω and 800 Ω are connected in series with a 7 volts battery. An ammeter of 10 Ω resistance is used to measure current.

(i) What will be the reading in the ammeter?

(ii) Similarly if a voltmeter of 10000 Ω resistance is used to measure the potential difference across the 600 Ω resistor, what will be the reading in the voltmeter?





4+6=10

(c) Define Fermi energy and show that for an electron gas at absolute zero temperature, the Fermi energy is given by

$$E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3}{8\pi}\frac{N}{V}\right)^{2/3}$$

where the symbols have their usual meanings. Estimate the numerical value of Fermi energy for copper taking number of electrons per unit volume as $8 \cdot 4 \times 10^{22}$ electrons/cm^3 and find Fermi temperature T_F. 10

8. (a) Write Bose-Einstein distribution function explaining every symbol. Derive Einstein's result on specific heat of solids explaining the assumptions made in the model. Discuss the low and high temperature limits of the predicted specific heat. In which limit, the Einstein formula fails to explain the experimental data? 10

(b) (i) The equation for an alternating current is $I = 42 \cdot 42 \sin(314t)$. Find the following :

Maximum value of current, Frequency, RMS value and Average value

(ii) A condenser of capacity 1 μF is first charged and then discharged through a resistance of 1 MΩ. Calculate the time in which the charge on the condenser will fall to 50% of its initial value.

(iii) Consider the displacement vector $\vec{D}$, given by

$$\vec{D} = (10xyz^2 + 4x)\hat{i} + (5x^2z^2)\hat{j} + (10x^2yz)\hat{k} \text{ nC/m}^2$$

Find the total charge enclosed in a cube of volume 10^{-9} m^3 located at the point (1, 2, 3). 5+5+5=15

(c) Explain Maxwell-Boltzmann formula for distribution of velocities of gas molecules at temperature T. What will be the formula for distribution of speeds? If $\bar{v}$, v_{rms} and v_m denote average speed, root mean square velocity and most probable speed, show that

$$\bar{v} : v_{rms} : v_m = \sqrt{\frac{8}{\pi}} : \sqrt{3} : \sqrt{2}$$

15

★★★