INDIAN FOREST SERVICE (MAIN) EXAM-2022

JHGT-U-PHY

PHYSICS
Paper – I

*Time Allowed : **Three Hours***

*Maximum Marks : **200***

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

*Answers must be written in **ENGLISH** only.*

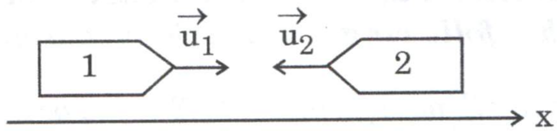
Useful Constants :

Electron charge (e)	$= 1{\cdot}602 \times 10^{-19}$ C
Electron rest mass (m_e)	$= 9{\cdot}109 \times 10^{-31}$ kg
Proton mass (m_p)	$= 1{\cdot}672 \times 10^{-27}$ kg
Vacuum permittivity (ε_0)	$= 8{\cdot}854 \times 10^{-12}$ farad/m
Vacuum permeability (μ_0)	$= 4\pi \times 10^{-7}$ T m/A
Velocity of light in free space (c)	$= 3 \times 10^{8}$ m/s
Boltzmann constant (k)	$= 1{\cdot}380 \times 10^{-23}$ J/K
Electron volt (eV)	$= 1{\cdot}602 \times 10^{-19}$ J
Planck constant (h)	$= 6{\cdot}626 \times 10^{-34}$ J s
Stefan constant (σ)	$= 5{\cdot}67 \times 10^{-8}$ W m^{-2} K^{-4}
Avogadro number (N)	$= 6{\cdot}022 \times 10^{26}$ $kmol^{-1}$
Gas constant (R)	$= 8{\cdot}31 \times 10^{3}$ J $kmol^{-1}$ K^{-1}
exp (1)	$= 2{\cdot}718$

SECTION A

Q1. (a) Consider a particle of mass m moving in a potential field of the form $V(\vec{r}) = V_0(x^2 + y^2)$, where V_0 is a constant. What are the conserved physical quantities for the particle ? *8*

(b) Two spaceships are approaching each other as depicted below.



Spaceship 1 is moving with a velocity $\vec{u}_1 = 0{\cdot}7c\,\hat{x}$, while spaceship 2 is moving with a velocity $\vec{u}_2 = -0{\cdot}8c\,\hat{x}$ as measured by the stationary observer on Earth. Here, c is the velocity of light in vacuum and $\hat{x}$ is the unit vector along the x-axis.

(i) What, according to Galilean addition of velocity, is the speed of spaceship 2 with respect to spaceship 1 (in terms of c) ?

(ii) What, according to relativistic addition of velocities, is the speed of spaceship 2 with respect to spaceship 1 (in terms of c) ?

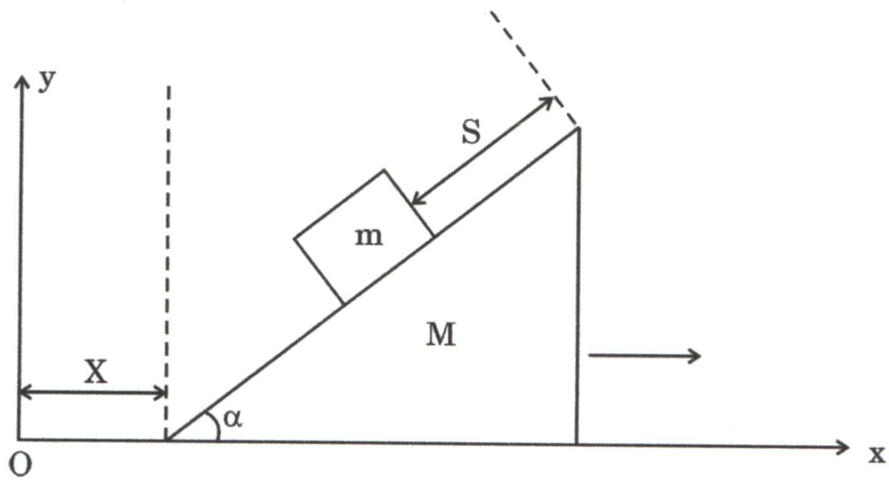
(iii) Spaceship 1 emits a light signal towards spaceship 2. The observer on spaceship 2 measures the frequency of the light signal to be 10^{16} Hz. What was the original frequency of the light signal ? *8*

(c) A tuning fork A produces 6 beats per second with another tuning fork B of frequency 508 Hz. When the prongs of A are loaded with a little wax, number of beats heard are 4 per second. Find the frequency of A before it was loaded with wax. *8*

(d) A transmission hologram is recorded in a medium of refractive index 1·15. If the spacing between the two consecutive maxima is 557 nm and the angle of incidence of reference and object waves from the reference axis (say z-direction) is 30°, find the wavelength of the light source. *8*

(e) The coherence length and the bandwidth of a light source is 3 km and 1×10^{-5} nm, respectively. Determine its wavelength and the pulse duration. *8*

Q2. (a) A small block of mass m slides without friction down a wedge-shaped block of mass M and opening angle α as depicted below.



The wedge-shaped block itself slides along the horizontal frictionless floor along the +ve x-direction. Find the horizontal acceleration $\ddot{X} = d^2X/dt^2$ of the wedge-shaped block by the Lagrangian method and using the displacements X and S of the blocks as generalized coordinates. *15*

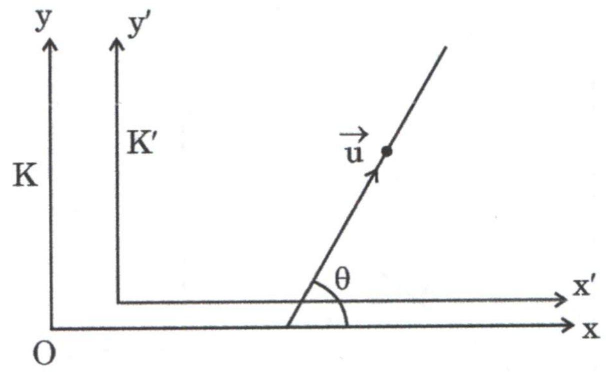
(b) Consider the motion of a particle of mass m in central force field $f(r) = -k/r^2$, where k is a constant and r is the distance from the force centre to the particle. Show that for the given system the vector

$$\vec{A} = \vec{p} \times \vec{L} - mk\frac{\vec{r}}{r}$$

is conserved. Using this vector and the fact that $\vec{A} \cdot \vec{L} = 0$, derive the orbit equation for the particle. *15*

(c) Consider the force-free motion of a symmetrical rigid body with z-axis as its axis of symmetry. Write down Euler's equations. Integrate them and obtain the solutions. On the basis of the obtained solutions show that the total angular velocity $\vec{\omega}$ of such a rigid body is constant in magnitude and precesses about the z-axis with a constant frequency, say Ω. Determine Ω. *10*

Q3. (a) A particle of rest mass m_0 is moving in a stationary frame K with a velocity $\vec{u}$ making an angle θ with the x-axis as shown in the figure.



Find the corresponding angle θ' made by the velocity vector of the particle with the x' axis in the frame K' that is moving with respect to K along the x-direction at a constant speed V. *15*

(b) What is forced oscillation ? Write its equation of motion and obtain the general solution. Discuss the condition for the resonance. *15*

(c) State Fermat's principle. Using this principle, establish the Snell's law of refraction. *10*

Q4. (a) In a Newton's ring experiment, explain the following : *10*

(i) Why are the fringes circular ?

(ii) Why is the central spot dark as seen by reflection ?

(iii) How can one obtain Newton's ring with bright centre ?

(b) What do you understand by missing orders in a double-slit diffraction pattern ? Obtain condition for missing orders in a double-slit Fraunhofer diffraction pattern, if the slit width and the separation between the slits are equal.

In a double-slit diffraction experiment, if the slit widths are 0·20 mm and they are 0·10 mm apart, find the missing orders. *15*

(c) Discuss the phenomenon of birefringence in a crystal. What do you understand by positive and negative crystals ?

If the refractive indices of a calcite crystal for E-ray and O-ray are 1·488 and 1·658, respectively, then calculate the thickness of the crystal for which it can work as

(i) quarter wave plate,

(ii) half wave plate. *15*

(Wavelength of the light is 5890 Å)

SECTION B

Q5. (a) Two dipoles 1 and 2 with dipole moments $\vec{p_1} = -5\ \hat{z}$ nC.m and $\vec{p_2} = 9\ \hat{z}$ nC.m, respectively, are located at points (0, 0, – 2) and (0, 0, 3), respectively. Find the potential at the origin. *8*

(b) The spectral energy density curve of the moon shows maxima at 14 μm. Estimate the temperature of the moon and the energy density of the lunar radiation. *8*

(c) A long straight wire of circular cross-section is made of non-magnetic material ($\chi_m = 1^\circ$ to a good degree of approximation). It is of radius a. The wire carries a current I which is uniformly distributed over its cross-section. Compute the energy per unit length stored in the magnetic field contained within the wire. *8*

(d) Estimate the temperature at which the root mean square speed of nitrogen molecules exceeds their most probable speed by 100 ms^{-1}. *8*

(e) Calculate the Fermi energy of aluminium at absolute zero. The density of aluminium is $2{\cdot}7 \times 10^3$ kg m^{-3} and its atomic weight is 26·98 kg $(k\ mol)^{-1}$. Show that the electron gas in aluminium is strongly degenerate. *8*

Q6. (a) The potential at the surface of a sphere (radius R) is given by $V_0(\theta) = k \cos 3\theta$, where k is a constant. Find the potential inside the sphere. *15*

(b) Consider an L-R-C series circuit with a 300 mH inductor, a 0·47 μF capacitor and a 500 Ω resistor. The source has terminal rms voltage V_{rms} =100 V and variable angular frequency ω. For what two values of the angular frequency, ω_1 and ω_2, is the rms current half the resonance value ? Also calculate the resonance width $|\omega_1 - \omega_2|$. *15*

(c) A wire in the form of a hexagon is just enclosed by a circle of radius 10 cm. If the current in the wire is 1 A, find the magnetic field at the centre of the hexagon. What would be the direction of the field if the current flows in the anti-clockwise direction ? *10*

Q7. (a) A relativistic charged particle moves in the space occupied by uniform and mutually perpendicular electric and magnetic fields $\vec{E} = E_0 \hat{x}$ and $\vec{B} = B_0 \hat{y}$, respectively. The particle moves rectilinearly along the z-direction. Find $\vec{E'}$ and $\vec{B'}$ in the reference frame moving translationally with the particle. *15*

(b) Consider a medium with an electric current density $\vec{J}$, permittivity ε and conductivity σ, which is described by Ohm's law $\vec{E} = \sigma \vec{J}$. Write down Maxwell's equations in such a medium. Using these equations, derive the wave equation for the electric field $\vec{E}$. Show that plane waves of the form

$$\vec{E} = \vec{E_0} \, e^{i(\omega t - kx)},$$

where $\vec{E_0}$ is a constant vector, are solutions of the wave equation but with complex k. Determine the explicit expression for k in terms of ε, permeability μ_0, σ and the angular frequency ω. *10*

(c) A solid copper sphere of mass 100 g kept at 300 K is suspended inside a closed chamber. The walls of the chamber are kept at 0 K. Assuming that the sphere acts as a blackbody, calculate the time required for its temperature to drop to half of the initial value. *15*

(Given that density of copper is $8{\cdot}96 \times 10^3$ kg m^{-3}, its specific heat capacity is 389 J kg^{-1} K^{-1})

Q8. (a) In the case of a gas obeying the equation of state $\frac{Pv}{RT} = 1 + \frac{\beta}{v}$, where β is a function of T only, find the expression of the heat capacity at constant volume. *15*

(b) Two Carnot engines C_1 and C_2 operate in series, Engine C_1 absorbs heat at T and rejects heat to a sink at temperature 300 K. Engine C_2 absorbs $\frac{1}{4}$th of the heat rejected by engine C_1 and rejects heat to the sink at 200 K. If the work done in both the cases is the same, find the temperature T. *15*

(c) Liquid ^{4}He has normal boiling point at 4·2 K. It is observed that it boils at 1·2 K at a pressure of 1 mm of Hg. Calculate the average latent heat of vaporization of ^{4}He in this temperature range.
(Given : Density of Hg = $13{\cdot}6 \times 10^3$ kg m^{-3}). *10*