STATISTICS

Paper – II

Time Allowed : **Three** Hours

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

*Answers must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION A

Q1. (a) Explain different causes of variation in the quality of manufactured products. Also, what are the criteria for detecting lack of control in $\overline{X}$ and R charts ? *8*

(b) Explain item-by-item, single sample sequential sampling plan. Also explain lot-by-lot inspection plan for variables. *8*

(c) Explain a parallel system and its parameters. *8*

(d) Stating the primal – dual relationship in linear programming problems, write the dual of the following primal problem :

Maximize $z = 2x_1 - x_2 + x_3$

subject to

$$2x_1 + 3x_2 - 5x_3 \geq 4$$

$$-x_1 + 9x_2 - x_3 \geq 3$$

$$4x_1 + 6x_2 + 3x_3 \leq 8$$

$$x_1, x_2, x_3 \geq 0.$$ *8*

(e) Define a Markov chain. What do you understand by a Markov chain of order k ? Given the following T.P.M. of a Markov chain with states (0, 1),

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{array}$$

find the two-step transition matrix and hence find $p_{11}^{(2)}$. *8*

Q2. (a) A convenient notation for summarizing the characteristics of a queuing model is generally given by the format : (a/b/c) : (d/e). What are the characteristics of the queue which these letters show ?

Find the distribution of a single-server queuing model with finite queue size, Poisson arrival and service rates, if the services are provided with first come, first served basis. *15*

(b) Mention different control charts for attributes. Also describe the method of constructing p-chart. 15

(c) What do you mean by the Transition Probability Matrix (T.P.M.) of a Markov chain ? Mention some of its properties.

Let $\{X_n\}$ be a Markov chain with state space $\{0, 1, 2\}$, initial probability vector $P(0) = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$ and one-step transition matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix} \end{array}$$

(i) Compute $P[X_0 = 0, X_1 = 1, X_2 = 1]$.

(ii) Show that $P[X_1 = 1 \text{ and } X_2 = 1 \mid X_0 = 0] = p_{01}p_{11}$.

(iii) Compute $p_{01}^{(2)}$. 10

Q3. (a) Differentiate between maintainability and availability. Also derive the expression for maintainability. 15

(b) Explain double sampling plan by attributes and compute ASN, ATI and OC curve for the same plan. 15

(c) Express a Transportation Problem (T.P.) in the form of a linear programming problem. Show that in a T.P. with m ports and n destinations, the number of basic variables will be m + n – 1. 10

Find the optimal solution of the following T.P. :

Destinations

		D_1	D_2	D_3	D_4	D_5	D_6	Availability
Ports	O_1	2	1	3	3	2	5	50
	O_2	3	2	2	4	3	4	40
	O_3	3	5	4	2	4	1	60
	O_4	4	2	2	1	2	2	31
Demand		30	50	20	40	30	11	

Q4. (a) Define a Linear Programming Problem (LPP) with m constraints and n variables x_i $(i = 1, 2, ..., n)$; $(m < n)$. Show that the set of all feasible solutions to an LPP is a convex set.

Find the solution of the following LPP :

Minimize $z = 5x_1 - 4x_2 + 6x_3 + 8x_4$

subject to

$$x_1 + 7x_2 + 3x_3 + 7x_4 \leq 46$$

$$3x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$2x_1 + 3x_2 - x_3 + x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0.$$ *15*

(b) Define renewal process. For $k \in N$ and $t \geq 0$, show that $\{N_t \geq k\}$, if and only if $\{S_k \leq t\}$. *15*

(c) What is a general inventory model ? Describe the various costs involved in it. Obtain the expression of optimum order quantity in an inventory model with constant rate of demand, instantaneous order replacement and no shortage. *10*

SECTION B

Q5. (a) Define time series data and explain moving average for a multiplicative model. *8*

(b) Describe the problem of 'multicollinearity' in econometrics and explain in brief how you will detect it. *8*

(c) Derive an algebraic expression relating the probability of a person dying between the age of x and (x + 1), q_x to the force of mortality, μ_x. *8*

(d) Distinguish between stable and stationary populations. *8*

(e) What are σ and standard scores ? The fifth grade norms for a reading examination are Mean = 60 and SD = 10; for an arithmetic examination Mean = 26 and SD = 4. A student scores 55 on the reading and 24 on the arithmetic test. Compare his σ scores. In which test is he better ? *8*

Q6. (a) For the following data, construct the cost of living index for the year 2013 (Base 1990 = 100) using the method of weighted price relatives : *15*

Item	Unit	Price (1990)	Price (2013)	Weight
A	Kg.	50	75	10%
B	Litre	60	75	25%
C	Dozen	200	240	20%
D	Kg.	80	100	40%
E	One pair	160	200	5%

(b) State the problem of identification. Explain with suitable notation, the rank and order conditions for identifiability.

Identify the following system : 15

$$y_1 = 3y_2 - 2x_1 + x_2 + U_1$$

$$y_2 = y_3 + x_3 + U_2$$

$$y_3 = y_1 - y_2 - 2x_3 + U_3$$

(c) Compute the standardised death rates for two countries A and B from the following data and compare them : 10

Age group (years)	Death Rate per 1,000		Standardised Population (in lakhs)
	Country A	Country B	
0 – 4	20·0	5·0	100
5 – 14	1·0	0·5	200
15 – 24	1·4	1·0	190
25 – 34	2·0	1·0	180
35 – 44	3·3	2·0	120
45 – 54	7·0	5·0	100
55 – 64	15·0	12·0	70
65 – 74	40·0	35·0	30
75 and above	120·0	110·0	10

Q7. (a) Name any two central statistical organizations in India and explain their functions. 15

(b) Given the life table of three men A, B and C aged 90, 91 and 92 years respectively as follows :

Age x	l_x
90	16090
91	11490
92	8012
93	5448
94	3607
95	2320
96	1447
97	873
98	590
99	98
100	0

where l_x = Number living at age x.

Find the probability that

(i) A, B and C will be alive in two years time (i.e., at the end of two years)

(ii) All will be dead within two years, and

(iii) C will be alive for 6 years' time. *15*

(c) Explain Autoregressive model and state its stationary invertible property. *10*

Q8. (a) What is logistic curve ? Explain the method of three selected points for fitting the logistic curve to a population data. *15*

(b) Consider the general linear stochastic model :

$$Y = X\beta + U$$

where Y is an n × 1 vector of observations of the dependent variable, X is an observed n × k matrix of rank k, β is a column vector of k unknown parameters and U is an n × 1 disturbance vector such that

$$E(U) = 0 \text{ and } E(UU') = \sigma^2 I_n.$$

Obtain the expressions for the least squares estimator and its variance – covariance matrix. *15*

(c) Define reliability and validity of a test. Discuss the effect of lengthening of a test on its reliability and validity. How are validity and reliability related to each other ? *10*