I.F.S. EXAM-(M) 2018

STATISTICS

Paper – II

Time Allowed : **Three** Hours

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

*Answers must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION A

Q1. (a) What are the situations that depict the lack of control in $\overline{X}$ and R charts ? *8*

(b) What are Military Standard Tables ? Explain its uses in Statistical Quality Control theory ? *8*

(c) Explain the concepts of Type I censoring and Type II censoring. Describe the situation of them arising either by design or due to experimental circumstances. *8*

(d) State the duality theorem in linear programming problem. Write the dual of the following primal problem : *8*

Minimize $z = 2x_1 + 3x_2 + 4x_3$

subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2 \geq 0$, x_3 unrestricted in sign.

(e) Describe different features of a transition probability matrix in reference to a Markov chain. Given the following transition matrix of a Markov chain with three states 1, 2 and 3 and with initial probability distribution $\pi_0 = [0{\cdot}7, 0{\cdot}2, 0{\cdot}1]$, find the value of *8*

$$P[X_3 = 2, X_2 = 3, X_1 = 3] : \begin{bmatrix} 0{\cdot}10 & 0{\cdot}50 & 0{\cdot}40 \\ 0{\cdot}60 & 0{\cdot}20 & 0{\cdot}20 \\ 0{\cdot}30 & 0{\cdot}40 & 0{\cdot}30 \end{bmatrix}$$

Q2. (a) Explain the meanings of (i) basic solutions, and (ii) feasible solutions in a linear programming problem with m conditions and n variables. Using simplex method, solve the following linear programming problem : *15*

Maximize $z = 3x_1 + 2x_2 + 5x_3$.

subject to $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_2 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$.

(b) What are the different control charts for attributes used in Industrial Inspection of manufactured units ? Also calculate the control chart for the number of defects and comment whether the process is under control or not based on the following data : *15*

Piece No. :	1	2	3	4	5	6	7	8	9	10
Number of defects :	4	3	6	3	0	1	3	5	7	8

(c) Describe classification of states in a Markov chain. What is an n-step transition probability ?

Prove the following Chapman-Kolmogorov equation for transition probabilities :

$$p_{ij}^{(n+1)} = \sum_{k} p_{jk} \, p_{kj}^{(n)},$$

where symbols have their usual meanings. *10*

Q3. (a) Define the terms of Reliability function and Failure rate function of a random variable denoting life time of a component. Establish the relation between them if any exist. Also prove that

$$\int_{0}^{\infty} h(t)\, dt = \infty$$

where h(t) is the failure rate function. *15*

(b) A food company puts mango juice into cans advertised as containing 200 ml of the juice. Quantity of the cans after filling for 10 samples of 4 cans each are taken by a random method at an interval of 60 minutes. Following presented below are the excess over 200 ml in each can. Construct an $\overline{X}$-chart to control the volume of mango juice for filling. (Table Value of A_2 for n = 4 is 0·729). *15*

Sample No.	Can 1	Can 2	Can 3	Can 4
1	15	12	13	20
2	10	8	8	14
3	8	15	17	10
4	12	17	11	12
5	18	13	15	4
6	20	16	14	20
7	15	19	23	17
8	13	23	14	16
9	9	8	18	5
10	6	10	24	5

(c) What do you mean by an assignment problem (AP) ? Describe the steps of the method for solving an AP.

A car rental service has a surplus of one car in each of the cities 1, 2, 3, 4, 5 and 6 and a deficit of one car in each of the cities 7, 8, 9, 10, 11 and 12. The distance (in kilometres) between cities with a surplus and cities with a deficit are shown in the matrix below. Work out an optimal assignment of surplus cars and the corresponding total distance to be travelled. *10*

		To					
		7	8	9	10	11	12
From	1	41	72	39	52	25	51
	2	22	29	49	65	81	50
	3	27	39	60	51	32	32
	4	45	50	48	52	37	43
	5	29	40	39	26	30	33
	6	82	40	40	60	51	30

Q4. (a) What is a multi-channel queueing problem ? Deduce difference-differential equations for the (M/M/K) : (∞/FIFO) queueing system and obtain the steady-state solution for the system size. *15*

(b) Develop hazard functions when the life pattern of a system was described by *15*

(i) Exponential,

(ii) Weibull, and

(iii) Lognormal distribution.

(c) Describe a two-person zero sum game and in this context explain the terms (i) pay-off matrix, (ii) saddle point, and (iii) mixed strategies.

Let f(i, j) be a real-valued function and be defined whenever $i \in A, j \in B$.

Suppose that both $\max_i \min_j f(i, j)$ and $\min_j \max_i f(i, j)$ exist. Then prove that a necessary and sufficient condition that $\max_i \min_j f(i, j) = \min_j \max_i f(i, j)$ is that the function f possesses a saddle point. *10*

SECTION B

Q5. (a) What are the components of time series data ? Explain least square method of fitting a trend line. *8*

(b) What is meant by Identification problem in simultaneous equation models ? Distinguish between exactly identified, over identified and unidentified. *8*

(c) With usual notations, explain abridged life table columns and establish the relationship between Age Specific Death Rate (${}_nM_x$) and Life Table Death Rate (${}_nq_x$). *8*

(d) Explain the method of collecting demographic data using the method of registration, stating its uses and limitations. *8*

(e) What are T-scores and standard scores ? Mention the uses of T-scores and compare it with standard scores. *8*

Q6. (a) Explain time reversal and factor reversal tests. Show that the Marshall-Edgeworth index number lies in between Laspeyres' and Paasche's index numbers. *15*

(b) What is Heteroscedasticity ? Explain the following methods of detecting heteroscedasticity : (i) Graphical method, (ii) Park test, and (iii) Glejser test. *15*

(c) Explain the need of using standardized death rates. Also describe the method of computing standardized death rate using indirect method. *10*

Q7. (a) Explain the role of CSO and NSSO, the official statistical organisations in India, in collecting statistics. *15*

(b) Define Total Fertility Rate (TFR) and Net Reproduction Rate (NRR). Explain their importance in context of population growth. *15*

(c) Explain different phases of Box-Jenkins methodology for time series data analysis using flow chart. *10*

Q8. (a) Define and distinguish between stationary and stable populations. Explain Gompertz population growth model and state your comment. *15*

(b) What is Multicollinearity ? Give indicators which help in detecting multicollinearity. Also explain any two remedial measures to minimise multicollinearity. *15*

(c) Explain the concepts of reliability and validity of scores in educational and psychological experiments. Mention the effect of lengthening a test upon reliability and validity. Test A has a reliability coefficient of 0·70 and a correlation of 0·40 with the criterion p. What would be the correlation of test A with the same criterion, if the test were tripled in length ? *10*